Sraffa's 'Reduction' of the Prices of Basics

Contribution to the ESHET Conference 2021in Sofia

Paper forthcoming in Structural Change and Economic Growth

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Summary

Piero Sraffa, in his *Production of Commodities by Means of Commodities*, used his so-called Standard Commodity as a tool for his exposition and for his derivation of Standard Prices. The paper shows that the reduction to dated quantities of labour, in which this exposition culminates, implies a diminution of the indirect labour inputs to any commodity over time that exhibits certain regularities, if the commodities are basics, and this helps to explain why the paradoxes of capital theory such as reswitching and reverse capital deepening are rare.

Key Words: Capital theory, Reduction, Sraffa

JEL codes: B24, B51, , D24, D57, E25

I have known Luigi Pasinetti for 50 years. We first met in spring 1969 in Cambridge, when he was teaching at King's College and I was a visitor to the Faculty of Economics with a grant from the Swiss National Foundation. I had a diploma in mathematics, physics and philosophy, but was only on my way to becoming an economist. My supervisor in Basel, Gottfried Bombach, had recommended me to James Meade, who was ready to act informally as my supervisor abroad. Our discussions developed well as long as I had to write on Meade's work on the theory of growth (Meade 1968), but then I discovered the book by Jacob Schwartz *Mathematical Methods in Analytical Economics* (Schwartz 1961) and had begun to work on Piero Sraffa's *Production of Commodities by Means of Commodities* (Sraffa 1960).

I had become acquainted with Meade in the Common Room. He gave me the advice not to begin my work as an economist with capital theory; it was "highbrow" he said, and when I persisted, I was left without informal supervision. I met Pasinetti who confronted me some months later in the presence of Sraffa with Manara's article on Joint Production. They asked me what I thought about it. Manara (1968) had given an example of a joint production economy without a standard commodity. They said that the example was meaningless because one sector in this two-sector model needed less of both inputs and produced more of both outputs than the other process. Why was the inefficient process not discarded? I came back a few days later and explained that I thought that the inefficiency could not be ruled out, for if the less efficient process used less labour, it could be as profitable as the more efficient one at a high wage rate and would turn inefficient only if the rate of profit was high. They had to accept this possibility, but Yoann Verger and Ajit Sinha still defend Sraffa's first reaction to Manara (see our controversy in Sinha 2021). I began to write my thesis on the theory of joint production (Schefold 1971).

Pasinetti was heavily involved in the capital debate, which had culminated in the disproof of Levhari's (1965) assertion that reswitching could be ruled out, if alternative methods were given for single product systems that were basic. One example for reswitching in the case of non-basic systems is in Sraffa's book (Sraffa 1960, pp. 37-8). Pasinetti was the first to provide a numerical counterexample to Levhari's faulty proof

(Pasinetti 1966). He would also defend the critique against Solow (Pasinetti 1969).

But it soon became clear that he wanted to develop the classical and Keynesian approach as a theory capable of application in a normative sense: conditions could be derived for a disaggregated industrial economy to grow at full employment with demand changing with the growth of incomes according to Engel curves and with productivity growing at a regular pace, characteristic for each integrated sector of the economy (Pasinetti 1981). A complex and difficult model resulted which Pasinetti later would deskill by assuming that the commodities of each sector were produced by means of simple labour alone (Pasinetti 1993). Simplifying models in order to attack grander questions was one of the Pasinetti's devices to reach relevance – *reculer pour mieux sauter* was a French idiom which he would quote on such occasions. Capital theory in its full complexity was an obstacle, for it implied problems not only for neoclassical, but also for the Keynesian theories of steady growth that were developed during the 1960s and 1970s.

The simplest example of how capital theory may help to explain problems of the neo-Keynesian theory of growth is as follows. According to the stylized facts of growth theory, the rate of growth, the rate of profit and the distributional shares remain constant, and this constancy may be explained by means of the post-Keynesian theory of distribution, with $r = g/s_c$, where r is the rate of profit, g the rate of growth, s_c the propensity to save out of profits and s_w the propensity to save out of wages, which we assume to be zero (Kaldor 1978, Robinson 1968). The technique that is given at any moment then must be represented by a straight line turning around a constant maximum rate of profit, as in Diagram 1; the real wage rate at $\bar{r}$ then increases regularly with the growth of productivity, as represented at $r = 0$. If the wage curves were not straight, the capital coefficient would fluctuate during the growth process, and if there is a switch point between two techniques which come into existence successively, with the switch point lying in between zero and the actual rate of profit, a fall in the capital coefficient and in the capital labour ratio is implied if the process takes place at full employment. The requirement for capital then diminishes against expectations so that a crisis of overproduction may result. The problem was discussed in Schefold (1979).

Post-Keynesian theory admits the possibility of such disturbances but the theory of capital, as it was developed at the time, made no prediction as to how frequent such disturbances might be. On the one hand, Cambridge economists like Joan Robinson and Nicholas Kaldor developed the steady state theory according to Diagram 1 (Kaldor 1978; Robinson 1968). On the other, they would speak of Wicksell effects, of reswitching and reverse capital deepening, when it was a matter of criticizing neoclassical theory (Robinson 1973-9). If these phenomena were to be taken seriously as a critique, what did that mean for the Keynesian theory itself? The post-Keynesians admitted the possibility of disturbances of the growth process, but why were they not more frequent? On this, their theory remained silent.

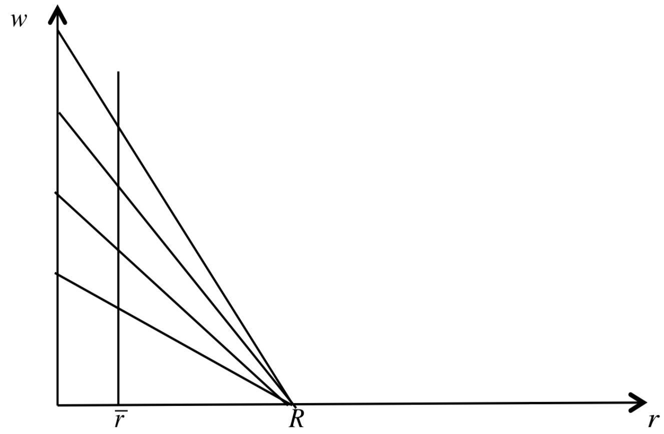

Diagram 1: The stylised facts of postkeynesian theory must be represented by linear wage curves showing the growth of labour productivity of labour, with a constant maximum rate of profit, since the capital-output ratio is supposed not to change either with distribution or over time.

The third part of Sraffa's book created the impression that any change in distribution would induce a multiplicity of switches and that the individual wage curves which would appear at any given rate of profit on the envelope could be of any curvature, provided only they were falling (Sraffa 1960, Part III).

Now it has turned out that the wage curves will not exhibit strong curvatures, if the technology exhibits random properties; insofar, the existence of an approximate surrogate production function does not seem to be all that unlikely after all (Schefold 2013a). A second result, however, redresses the balance in favour of the critique, in that the number of techniques appearing on the envelope is much smaller than the number of potential techniques. If there are n sectors of the economy, and in each sector m different methods are available, the number of potential techniques increases according to the formula $m^n = s$, where s is the number of techniques. But most of the wage curves corresponding to these techniques will remain below the envelope, and the number of techniques that one may expect to appear on the envelopes (the number of wage curves making up the envelope) will be equal to $\Omega = \ln s = n \ln m$, where $\ln$ denotes the natural logarithm (Schefold 2013b). This argument has been developed considerably in (Kersting and Schefold 2021).

We cannot discuss the details of the randomness assumptions and certain modifications here. Suffice it to say that our theoretical reasoning suggests that each individual technique will be characterised by an individual wage curve, which will exhibit a Wicksell effect. Neoclassical and anti-neoclassical Wicksell effects will be about equally likely, but they will not be very large. As one moves down the envelope, the wage curves appearing on it will not exhibit Wicksell effects that alternate, but they will come in groups so that the envelope itself will in part show neoclassical, in part anti-neoclassical Wicksell effects. Switches will not be so frequent that the neoclassical representation of the change of technique as a continuous process, as distribution varies, would seem justified. On the contrary, the small changes of the rate of profit and the wage rate that may be observed in reality (for instance during the cycle) may be associated with only a small number of substitutions, if any, and this picture is confirmed by the available empirical data. As a matter of fact, it was first discovered empirically that the wage curves are usually not far from linearity and that the number of switch-points on the envelope is relatively small (Han/Schefold 2006), and only later the assumptions about randomness were introduced as explanations of this fact.

The consequences, which follow from these observations, may be illustrated by means of the Wicksell process. According to Wicksell and the Austrian tradition, an artificial lowering of the rate of interest will induce entrepreneurs to move to a higher capital intensity so that they are at full employment forced to invest more; they thus create an artificial boom with forced saving that ultimately will result in a crisis. There is a truth in this. In a Keynesian perspective, following Minsky (1976), the lowering of the rate of interest may indeed lead to a boom in the form of an increase in effective demand, hence to an increase in employment, and if previously there had been full employment, foreign labour will now be attracted to this economy. If the boom lasts, it will lead to a Minsky moment in that entrepreneurs will resort to methods of finance that become increasingly risky. First only a few, soon more firms follow; some fail until trust collapses and the crisis ensues. The difference with respect to the Austrian description of the matter consists in two aspects: On the one hand, the emphasis is on demand and employment in the Minsky case, not on prices and inflation. On the other hand, there is no presumption in the Keynesian perspective that there is a change of the capital labour ratio. The reduction of the interest rate may lead to more investment, but not to a substitution of capital for labour. The prospects for lengthening the boom, provided measures against extreme risk-taking are instituted, will then be better.

The change in the orientation of capital theory may thus help to bring capital theory and the post-Keynesian theory of growth together (Schefold 2021). Such a desire to increase the realism is also visible in Pasinetti's work. In his later writings, he abstracts from the more pathological forms of paradoxes in the theory of capital and analyses the conditions under which full employment growth with technical progress may be made possible (Pasinetti 2007, Book Three).

It should also be mentioned that much research has been undertaken to represent the behaviour of prices of production or normal prices on the basis of empirical data, especially under the guidance of Anwar Shaikh (e.g. Shaikh et al. 2020); a leading early example is Bienenfeld (1988). He uses functions, which are quadratic in the rate of profit, to approximate the correct prices. The approximations are actually equal to the correct prices, if the rate of profit is zero or at its maximum; in-between, the approximation turns out to be good in applications. The approximation in itself, however, yields no explanation of why it is successful. To base it on parameters capable of interpretation is not yet an explanation. To take a simpler example: prices of production can be found to be close to labour values. It can be shown that this is possible, if and only if known parameters, the organic compositions, are uniform. But that does not tell why we encounter organic compositions that are close to uniform in reality, after having satisfied ourselves in the theory that wide deviations from uniformity would be possible. Hypotheses about randomness help to get deeper. The evolution of technology might be a random process, at least to some extent. But what we shall do here is the opposite: we question the theoretical preconception that relative prices somehow “must” vary dramatically, if the rate of profit changes.

We may then ask ourselves, what in Sraffa made us believe in the likelihood of the strong paradoxa like reswitching. That they exist is beyond doubt, but why did we believe that they might occur often? There were several reasons according to my recollection. I here want to discuss Sraffa’s ‘Reduction to dated quantities of labour’, with the application to the example of “wine” and “oak-chest”, where a numerical example for reswitching is given. However, Sraffa presents it in a curious combination with a basic system, which allows constructing the corresponding standard commodity, while “wine” and “oak-chest” are non-basics. It is by now quite obvious that examples with reswitching and other paradoxes can easily be constructed in the case of non-basics, but to what extent can this be extended to basics? This is a question, which early readers like Samuelson and Levhari asked. Non-basics yielded illustrative examples in that phase of the analysis. Multiple rates of return in Austrian models had been discussed already by Irving Fisher and Böhm-Bawerk. But the essential critique of capital concerns basic systems. We know that often “non-basics” become basics in the real world at a realistic level of analysis (ice-cream is supplied to hotels, which are inputs to the administration of large firms, etc.), and important capital goods are basics. I try to answer our problem by analysing a difference in the ‘Reduction’ between non-basics and basics that seems so far to have been overlooked.

Consider the ‘Reduction’. Prices and the wage rate are expressed in terms of the standard commodity so that the wage rate equals $w = 1 - r/R$. All costs are based on past labour, to the extent that this labour, embodied in commodity production, contributes indirectly to present production. $\mathbf{L}_t$ is the vector of past labour, expended

t periods ago, which enters present production in each line of industry indirectly. The proceeds from production can be invested and yield profit according to the ruling rate in each period. This investment may be repeated up to the present, so that the expenditure of labour $\mathbf{L}_t$ leads to a wage cost t periods ago of $w\mathbf{L}_t$, and this wage cost must be multiplied by $(1+r)^t$ because of the possibility of reinvestment, in order to get the total present cost of having expended $\mathbf{L}_t$; this is $w(1+r)^t\mathbf{L}_t$. The labour expended t periods ago, $\mathbf{L}_t$, is embodied via commodity production in the present product, so that $\mathbf{L}_t = \mathbf{A}^t\mathbf{l}$, assuming that the technique, prices, the wage rate and the rate of profit stay constant. The standard prices are therefore given by

$$\mathbf{p} = \sum_{t=0}^{\infty} w(1+r)^t \mathbf{L}_t = \sum_{t=0}^{\infty} (1-r/R)(1+r)^t \mathbf{A}^t\mathbf{l} \tag{1}$$

We here presuppose single production. The square input-output matrix is $\mathbf{A}$ is assumed to be semi-positive, indecomposable and primitive. The inputs a_{ij} represent the amount of commodity *j* to the production in industry i of one unit of commodity i (1). The formula, for which we have given the traditional intuitive reasoning, results mathematically from the price equations

$$\mathbf{p} = (1+r)\mathbf{A}\mathbf{p} + w\mathbf{l} \tag{2}$$

by expanding the corresponding solution for $\mathbf{p}$

$$\mathbf{p} = w\left(\mathbf{I} - (1+r)\mathbf{A}\right)^{-1}\mathbf{l}$$

using the expansion of the matrix inverse and normalizing prices by the standard commodity $\mathbf{q}(\mathbf{I}-\mathbf{A})$ with the normalization $\mathbf{ql} = 1$ where $(1+R)\mathbf{qA} = \mathbf{q}$. For a detailed exposition after motivation for the choice of the standard commodity see Schefold (1986). Taking the standard commodity as the numéraire means $\mathbf{q}(\mathbf{I}-\mathbf{A})\mathbf{p} = 1$ for all rates of profit between 0 and R. These prices are equal to labour values for $r = 0$.
If the capital-labour ratios are different in the different industries, therefore if we do not happen to have equal organic compositions of capital and if therefore prices are not equal to labour values at all rates of profit, relative prices must change as the rate of

profit increases, but the question is how big that change is going to be. Sraffa seems to have been of the opinion that relative price changes could be very great, because he analysed them in a special way. He split the terms of formula (1) into two components

$$f_t = \left(1 - r/R\right)\left(1 + r\right)^t \tag{3}$$

and

$$\mathbf{L}_t = \mathbf{A}^t\mathbf{l} \tag{4}$$

therefore into the polynomial (3), which reflects the influence of the rate of profit, and the vector of indirect labour of past period t. The product $f_t L_t^i$ is called 'labour term' by Sraffa (1960, p. 37): it is the 'constituent element' of the price of commodity i. He showed in a diagram, which we reproduce below as Diagram 2, that the polynomials f_t exhibit an increasingly sharp maximum for $t > 5$ between 0 and R as a result of opposed trends: on the one hand, the diminution of the wage rate, which tends to zero as r approaches R, and on the other hand the growing influence of accumulated profits over many periods that are expressed in the geometric function. Sraffa had the curves f_t drawn, multiplying them with quantities of labour that were the smaller, the higher t. In fact, equation (4) shows that the quantities of labour will diminish in each sector of the economy with t, but Sraffa published no attempt to determine at what rate the components of the vector $\mathbf{L}_t$ would diminish and to which amounts; he simply assumed diminishing quantities of labour "so as to keep the curves within the page" (Sraffa 1960, p. 36): an unusually funny way of putting it on his part. If now the different components of $\mathbf{L}_t$ diminished at different rates, different maxima of the polynomials f_t would dominate the price movement at different rates of profit for different commodities, and thus the 'terms' would explain the drastic movements of relative prices with distribution that constitute the core of Sraffa's critique of neoclassical theory.

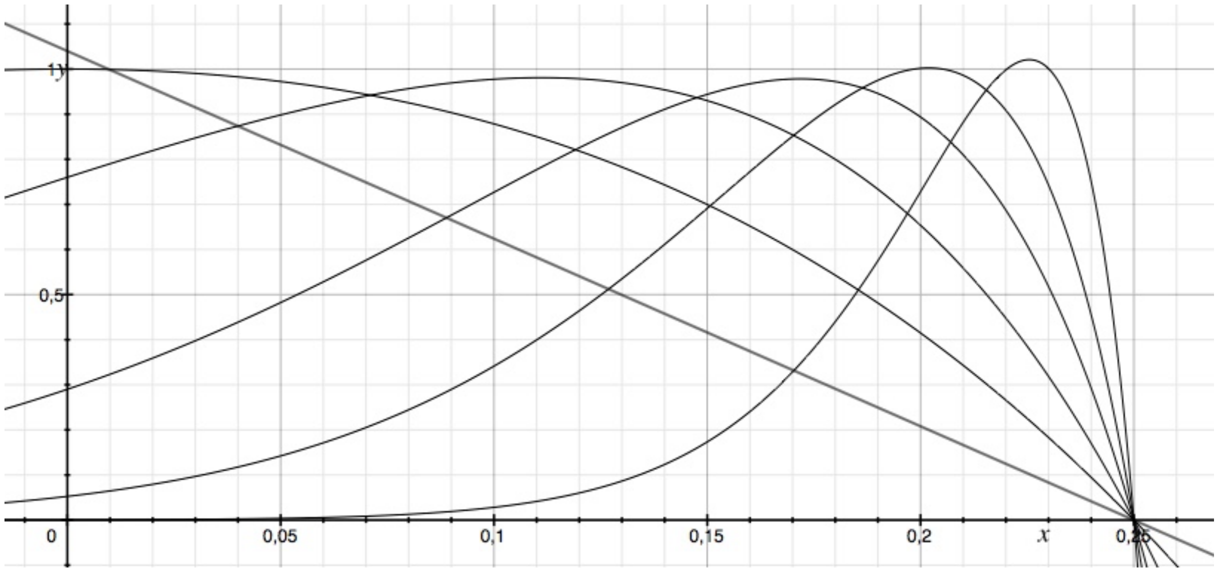

Diagram 2, showing the polynomials (3) with Sraffa's chosen values for the labour inputs that stand for the labour inputs (4). Diagram 2 thus duplicates the diagram in Sraffa (1960, p.36). Sraffa explains the diagram in the legend as follows: "Variation in value of 'Reduction terms' of different periods [$L_n w(1+r)^n$] relative to the Standard commodity as the rate of profits varies between zero and R (assumed to be 25%). The quantities of labour (L_n) in the various 'terms', which have been chosen so as to keep the curves within the page, are as follows: $L_0 = 1.04$; $L_4 = 1$; $L_8 = 0.76$; $L_{15} = 0.29$; $L_{25} = 0.0525$; $L_{50} = 0.0004$."

In fact, Sraffa here proceeded directly to the one and only explicit and direct critique of neoclassical theory that can be found in his book. He formulated the "wine and oak-chest" example, assuming that wine was produced a few years ago by an intermediate amount of labour and then was left to mature until it was ready for sale, whereas the oak-chest was produced by means of a small amount of labour, expended a long time ago (the planting of the tree) and then, in the present, the tree was harvested and transformed into the chest. Wine and oak-chest were non-basics, the wage rate, by contrast, was assumed to diminish according to the standard system of the basics, and the relative price fluctuated in such a way that wine and oak-chest had the same value at two different rates of profit. If the two commodities represented different capitals, as alternatives to be used for the same investment, the following paradox followed: one capital was cheaper and therefore more profitable to be used as capital at a low rate of profit, the other would become more profitable at an intermediate level, and the use of the first would again appear to be the more profitable investment at a higher level.

This was reswitching, but demonstrated for non-basics – basics came in only via the assumption of the linear wage curve. Samuelson and his pupil Levhari then thought that this phenomenon could not occur if all commodities were basic and Levhari presented a non- reswitching theorem for basic systems, but this was a mistake. As we have already seen, Levhari's proof contained an error.

However, we have also seen that, in the meantime, it has been found that the paradoxes of capital theory are empirically rare and theoretically unlikely. If that is true, it must be possible to show that Sraffa's core argument, why relative prices of capital goods (basics) change drastically, must be modified. What is wrong with Sraffa's diagram, that here is reproduced as Diagram 2? The answer is clear. What is lacking in Sraffa's representation of prices by means of dated quantities of labour is an analysis of how they diminish over time.

In order to derive the 'weight' of past labour inputs theoretically (and because the result will then also be empirically relevant), we now assume that matrix $\mathbf{A}$ is diagonalisable. As is well known, $\mathbf{A}$ can then be written as

$$\mathbf{A} = \sum_{i=1}^{n} \mu_i \mathbf{x}^i \mathbf{q}_i ,$$

where $\mu_1 \dots \mu_n$ are the eigenvalues of $\mathbf{A}$, where $\mu_1 = dom\mathbf{A} > 0$, and the eigenvalues can be ordered according to modulus so that $\mu_1 > |\mu_2| \geq \dots \geq |\mu_n| \geq 0$. The first eigenvalue is strictly greater than the second, because the matrix is primitive (no assumption about randomness is made). The $\mathbf{x}^i$ are the right hand side and the $\mathbf{q}_i$ the left hand side eigenvectors of $\mathbf{A}$; they are normalized so that $\mathbf{q}_i \mathbf{x}^j = \delta_{ij}$. The first eigenvectors are the Frobenius eigenvectors. Therefore $\mathbf{q}_1 = \overline{\mathbf{q}}$ is proportional to the standard commodity and $\mathbf{x}^1 = \overline{\mathbf{p}}$ is proportional to the standard prices obtained at the maximum rate of profit.

As is also well known, one obtains the following formula for the powers of $\mathbf{A}$, using the fact that the matrices $\mathbf{x}^i\mathbf{q}_i$ are idempotent, that is, $\left(\mathbf{x}^i\mathbf{q}_i\right)^t = \mathbf{x}^i\mathbf{q}_i$. We can write

$$\left(\frac{1}{\mu_1}\mathbf{A}\right)^t = \sum_{i=1}^{n}\left(\frac{\mu_i}{\mu_1}\mathbf{x}^i\mathbf{q}_i\right)^t = \mathbf{x}^1\mathbf{q}_1 + \sum_{i=2}^{n}\left(\frac{\mu_i}{\mu_1}\right)^t\mathbf{x}^i\mathbf{q}_i\,,$$

so that $\left(1/\mu_1\right)^t\mathbf{A}^t$ tends to $\mathbf{x}^1\mathbf{q}_1 = \overline{\mathbf{p}}\overline{\mathbf{q}}$, as t tends to infinity. We have $\mu_1 = 1/\left(1+R\right)$;

we may therefore also say that all elements of the matrix $\mathbf{A}^t - \overline{\mathbf{p}}\overline{\mathbf{q}}/\left(1+R\right)^t$ tend to zero.

Hence we now may write for Sraffa's reduction to dated quantities of labour (1)

$$\mathbf{p} = \left(1-\frac{r}{R}\right)\sum_{t=0}^{\infty}\left(1+r\right)^t\mathbf{A}^t\mathbf{l}$$

$$= \left(1-\frac{r}{R}\right)\sum_{t=0}^{T}\left(1+r\right)^t\mathbf{A}^t\mathbf{l} + \left(1-\frac{r}{R}\right)\sum_{t=T+1}^{\infty}\left(\frac{1+r}{1+R}\right)^t\overline{\mathbf{p}}\overline{\mathbf{q}}\mathbf{l} + \mathbf{z}\,, \qquad (5)$$

where for each $\varepsilon > 0$ there is T such that $Max\left|z_i\right| < \varepsilon$.

We therefore find that the polynomials f_t, which Sraffa used, can for sufficiently large t be replaced by the polynomials

$$g_t = \left(1-\frac{r}{R}\right)\left(\frac{1+r}{1+R}\right)^t \qquad (6)$$

and we then can write for indirect labour simply

$$\mathbf{L} = \overline{\mathbf{p}}\overline{\mathbf{q}}\mathbf{l} \qquad (7)$$

The implication is that Diagram 2 now looks different. We have incorporated the regular reduction of the influence of labour according to formula (7) in the polynomial (6). Diagram 3 shows the functions g_t between $r = 0$ and $r = R$. We see that the maxima of the polynomials become smaller more quickly than Sraffa suggested by means of his numerical examples. The maxima, far from being beyond the page, are all below the wage curve, but they are at the same rates of profit as in the case of f_t. In order to determine these rates of profit, at which maxima of g_t can be found, we differentiate g_t

and set the result to zero:

$$0 = g_t^{'} = -\frac{1}{R}\left(\frac{1+r}{1+R}\right)^t + \left(1-\frac{r}{R}\right)t\frac{\left(1+r\right)^{t-1}}{\left(1+R\right)^t}$$

$$0 = -1 - r + \left(R - r\right)t$$

The maxima of the functions g_t in the relevant range are therefore to be found at r_t where

$$r_t = \frac{Rt-1}{1+t} \tag{8}$$

We note that there is no maximum for $t = 0$, and the first maxima are at rates of profit smaller than zero, if $0 < R < 1$ (Sraffa assumes in his example $R = 25\%$). The maxima are then to be found at increasing rates of profit; these rates of profit tend to R as t tends to infinity.

We now insert (8) in (6), in order to obtain the values of the maxima. This gives

$$g_t\left(r_t\right) = \left(1 - \frac{1}{R}\frac{Rt-1}{1+t}\right)\left(\frac{1+\frac{tR-1}{1+t}}{1+R}\right)^t$$

$$= \frac{1+R}{R}\frac{1}{1+t}\left(\frac{t}{1+t}\right)^t \tag{9}$$

It is obvious that the maxima tend to zero for t increasing to infinity. To describe this movement, we note that $\left(t/\left(1+t\right)\right)^t$ tends to the inverse of Euler's number e, as t tends to infinity; hence $g_t\left(r_t\right)$, essentially goes to infinity with $\left(\left(1+R\right)/eR\right)\left(1/\left(1+t\right)\right)$, therefore hyperbolically. The maxima, each at its $r_t < R$, are below the linear or standard wage curve, and not far above it, as Sraffa's drawing suggested.

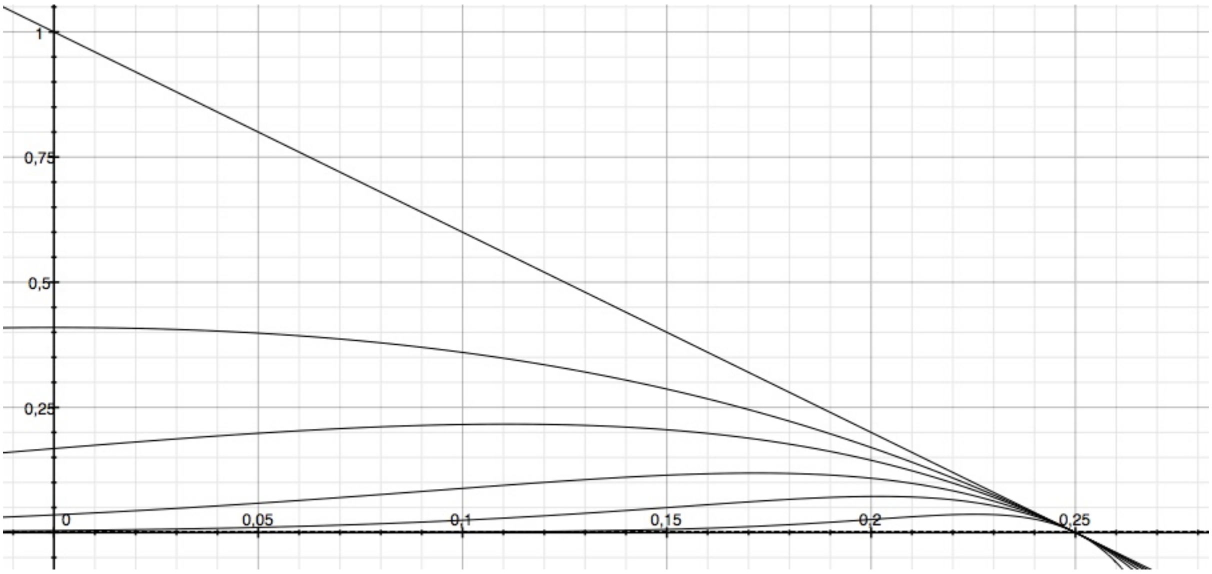

Diagram 3, showing the polynomials (6) for $t = 0,4,8,15,25,50$. Instead of Sraffa's chosen values for the labour inputs (L_n) quoted in the legend to Diagram 2, we here represent labour according to formula (6) by multiplying L_n by $(1+R)^{-t}$, where our t corresponds to Sraffa's n in Sraffa (1960, p.36). Evidently, the maxima of Sraffa's 'terms' are at the same rates of profit as in Diagram 2, but their peaks are well below the wage curve.

It follows from these considerations – and more intuitively from the simple comparison of Diagram 2 with Diagram 3 – that the diminution of the labour coefficients is so rapid that the maxima of the functions f_t or g_t cannot have an appreciable influence on relative prices for $t > T$. For clarification, we approximate the standard prices $\mathbf{p}$ by

$$\mathbf{p} \approx \sum_{t=0}^{T} f_t \mathbf{L}_t + \sum_{t=T+1}^{\infty} g_t \mathbf{L} = \mathbf{p}^I + \mathbf{p}^{II} \qquad (10)$$

We get for $\mathbf{p}^{II}$

$$\mathbf{p}^{II} = \sum_{t=T+1}^{\infty} g_t \mathbf{L} = \left(1 - \frac{r}{R}\right)\left(\frac{\left(1+r\right)^{T+1}}{\left(1+R\right)^{T+1}} + \frac{\left(1+r\right)^{T+2}}{\left(1+R\right)^{T+2}} + \ldots\right)\mathbf{L}$$

$$= \frac{R-r}{R}\left(\frac{1}{1-\frac{1+r}{1+R}} - \frac{1-\left(\frac{1+r}{1+R}\right)^{T+1}}{1-\frac{1+r}{1+R}}\right)\mathbf{L}$$

$$= \frac{1+R}{R}\left(\frac{1+r}{1+R}\right)^{T+1}\left(\bar{\mathbf{q}}\mathbf{l}\right)\bar{\mathbf{p}} \tag{11}$$

We may thus conclude that the prices $\mathbf{p}^{II}$, which reflect the influence of the higher terms in the infinite series given for standard prices in the reduction to dated quantities of labour (1), influence the *relative* prices only via the weight of *one* vector $\mathbf{L}$. Sraffa, because he did not realize how the labour inputs in the reduction to dated quantities of labour diminished as one went backwards in time, exaggerated the degree to which relative prices would change with distribution. While he was right that the paradoxa exist, the impression was created that they might occur more easily than they actually do.

Our critique has so far been concentrated on an analysis of the maxima of the polynomials in Sraffa's famous formula for standard prices (1): The indirect labour inputs fall so rapidly as one goes backward in time that the notion long past different indirect labour inputs could have an appreciable impact on relative prices in basic systems via the maxima turns out to be mostly an illusion. And there is a second reason why this influence cannot be large. The relative indirect labour inputs tend to become equal, as follows from (6) and (7). This convergence follows a regular pattern. It is worthwhile to look at this aspect more closely. To do so, we return to our discussion of the "wine and oak-chest" example. We saw that relative prices are what matters for the critique of capital theory. If we continue to regard the rate of profit as the exogenous variable, the wage rate drops from the picture in the consideration of relative prices. Let i and j denote two different commodities. Their relative price can then be written as

$$\frac{\mathbf{p}_i}{\mathbf{p}_j} = \frac{\hat{\mathbf{p}}_i}{\hat{\mathbf{p}}_j} = \frac{\mathbf{e}_i \sum_{t=0}^{\infty}\left(1+r\right)^t \mathbf{A}^t\mathbf{l}}{\mathbf{e}_j \sum_{t=0}^{\infty}\left(1+r\right)^t \mathbf{A}^t\mathbf{l}}$$

But this formula does not show well how relative prices change because of distribution. It is the advantage of Sraffa's standard prices that their formula renders the contrary effect of the wage rate and the rate of profit evident, which is due to the different capital intensities in different industries, both directly (in the industry, in which the commodity is produced) as well as indirectly (in the industries, in which the means of production are produced).

Hence, we write the standard prices in the form

$$\mathbf{p}_i = \sum_{t=0}^{\infty}\left(1-\frac{r}{R}\right)\left(1+r\right)^t\left(\mathbf{L}_t\right)_i$$

where

$$\left(\mathbf{L}_t\right)_i = \mathbf{a}_i\mathbf{A}^{t-1}\mathbf{l}$$

is the indirect labour contribution from period t to industry i. If one now assumes that the indirect labour contributions can be arbitrary numbers, as would be correct for non-basics, we may compare two commodities which have the same direct and indirect labour inputs in all periods except one, where one of the commodities has a larger labour input than the other. If the rate of profit, at which we compare the prices, happens to be r_t, the rate of profit, at which the polynomial f_t reaches its maximum, a given large difference between $\left(\mathbf{L}_t\right)_i$ and $\left(\mathbf{L}_t\right)_j$ will translate into a very large difference between $\mathbf{p}_i$ and $\mathbf{p}_j$. For example: If we compare two bottles of wine (assumed to be non-basics!) which require the same amount of direct and indirect labour at all periods, except at t, where the grapes are collected and brought to the press, a significant difference of price may result at r_t, if one of the bottles contains ordinary wine and the other ice-wine, for considerably more labour is required to select and collect the almost frozen grapes for ice-wine. Hence the price difference will be big, if the rate of profit is close to r_t and not so significant otherwise.

But if the commodities are basics, the differences in the expenditure of labour of many periods ago will average out, the more so, the larger is t. Mathematically speaking, the series $\mathbf{L}_0 = \mathbf{l}, \mathbf{L}_1, \mathbf{L}_2, \ldots$ is not only diminishing, but must exhibit regularities because the prices can be written as ratios of finite polynomials of degree n and $n-1$. The relative price can also be written as

$$\frac{\mathbf{p}_i}{\mathbf{p}_j} = \frac{\mathbf{e}_i \left(\mathbf{I} - (1+r)\mathbf{A}\right)_{Ad} \mathbf{l}}{\mathbf{e}_j \left(\mathbf{I} - (1+r)\mathbf{A}\right)_{Ad} \mathbf{l}}$$

where Ad denotes the adjoint of a matrix. The relative price is therefore the ratio of two polynomials of degree $n-1$. If the same relative price can be written also as the ratio of two infinite series, the coefficients of the series cannot all be arbitrary. To see this, it is not necessary to expound the mathematical theory fully, which is well known; an example suffices. Assume that the roots of the polynomials in the nominator and the denominator of the formula is describing a relative price are simple and different, so that each polynomial is the product of linear factors such as $1-x$ and $1+x$. If such a polynomial is in the nominator, we get for instance

$$\frac{1}{1-x^2} = \frac{1}{(1-x)} \frac{1}{(1+x)} = \left(1 + x + x^2 + \ldots\right)\left(1 - x + x^2 - \ldots\right) = 1 + x^2 + x^4 + \ldots$$

The polynomial is a product of linear factors and the infinite series result from the multiplication of simple geometric series. It must therefore be possible to identify the corresponding regularities in the series of the $\mathbf{L}_t$, but we cannot go more deeply into the analysis.

Enough has been said to demonstrate that one must not invent the $\mathbf{L}_i$ arbitrarily, if one wants to construct economically meaningful examples for the prices of capital goods, which are typically basics. The series of the $\mathbf{L}_t$ must be such that it could result from some input matrix $\mathbf{A}$ and some labour vector $\mathbf{l}$, with $\mathbf{A}$ and $\mathbf{l}$ fulfilling the usual conditions of semipositivity, indecomposability etc., and each $\mathbf{L}_t$ must result from $\mathbf{A}$ and $\mathbf{l}$ according to

$$\mathbf{L}_t = \mathbf{A}^t \mathbf{l} \qquad (12)$$

This is a requirement that follows from assumptions that are generally recognized as necessary and meaningful in the debate about capital theory. It is a postulate most typical for what one ironically calls "armchair economics". Plausible assumptions are made and implications are debated. The critics of neoclassical capital theory, who try to point out inconsistencies in theories with shared assumptions, must be consistent themselves. The question was: could reswitching, which had been shown to exist in the "wine and oak-chest" example, be extended to basics? It turned out that this was the case, but later it was found that this – and even reverse capital deepening – was empirically rare and theoretically unlikely. In the end, the examples we use to illustrate the relevance of theoretical assertions must not only be admissible according to a broad set of theoretical assumptions, but also plausible empirically. With the present paper we have demonstrated that the causes, which explain why these paradoxa are difficult to encounter, can be made visible with precisely that conceptual instrument that was used to provide the first example of reswitching: Sraffa's Reduction to Dated Quantities of Labour. It only has to be analysed more closely. Why did we not do it 50 years ago?

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(13.07.20 BS/ba, 30.08.21 BS)