How Macroeconomists Lost Control

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Abstract

This paper explores the history of the mathematical tools using feedback rules for macroeconomic stabilization policy from the perspective of the field of control. It describes the path which led mainstream macroeconomics to be the only field of research in all sciences using control which decided to promote stabilization with positive-feedback rule parameters that destabilize the dynamic system of the macroeconomic policy targets.

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1 Introduction

Aström and Kumar (2014) surveys the research field of control based on negative feedback. "*Feedback is an ancient idea, but feedback control is a young field... Its development as a field involved contributions from engineers, mathematicians, economists and physicists. It represented a paradigm shift because it cut across the traditional engineering disciplines of aeronautical, chemical, civil, electrical and mechanical engineering, as well as economics and operations research. The scope of control makes it the quintessential multidisciplinary field.*" (Aström and Kumar, 2014, p.3). The main idea of control is to use negative feedback rule in order to stabilize a dynamic system in these disciplines.

Kendrick (1976), Kendrick (2005) and Neck (2009) are detailed accounts of the history between economics and control. Levine (2008) surveys dynamic games and time inconsistency. Turnovsky (2011) surveys stabilization policies.

However, using positive-feedback policy rule parameters for macroeconomic stabilization in order to achieve determinacy, eliminate sunspots in order to "stabilize the economic system" is an established tool in mainstream macroeconomics since the mid-1990s. Macroeconomics is currently the only field of research initially using the tools of negative-feedback control which emancipated and chose to do the opposite with respect to the field of control. If the idea of positive-feedback rule parameter is wrong, from the point of view of robust control theory, positive-feedback stabilization policy is a dangerous idea advocating the opposite of what should be done.

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This paper sets an history of the mathematical tools using feedback rule for stabilization policy in macroeconomics. Section 2 presents a benchmark example of negative-feedback versus positive-feedback rule parameters. Section 3 presents an history of negative-feedback stabilization policy: simple rule (classic control), optimal policy with modern control, Stackelberg dynamic games (Ramsey optimal policy) and robust control. Section 4 presents the arguments for the ineffectiveness of stabilization policy. Section 5 describes how the positive feedback rule parameters theory emerged. Section 6 concludes with weaknesses of the positive feedback rule stabilization policy.

2 Positive versus Negative Feedback Rule Parameters

2.1 Transmission mechanism and proportional feedback rule

Consider the monetary policy transmission mechanism as a first order single input (policy instrument, nominal funds rate i_t) single output (inflation persistence targeting: inflation π_t is the only policy target) linear model, along with the effect of an exogenous auto-regressive variable. This variable is usually assumed to be not observable time series. Both the policy target and the policy instrument are written in deviation of their long run equilibrium values:

$$\pi_{t+1} = A\pi_t + Bi_t + Cz_t \text{ with } A > 0,\ C > 0,\ B \neq 0 \quad (1)$$

$$z_t = \rho z_{t-1} \text{ with } 0 < \rho < 1. \quad (2)$$

A proportional feedback rule is given by:

$$i_t = Fi_t + F_z z_t \quad (3)$$

Substituting the feedback rule in the transmission mechanism leads to "closed-loop" dynamics

$$\pi_{t+1} = (A + BF)\,\pi_t + (C + BF_z)\,z_t \quad (4)$$

2.2 Lucas' critique

This model presented in the previous section corresponds to a linear scalar model of the one presented by Lucas (1976) in the last pages of the Lucas' critique paper. The persistence parameter $(A + BF)$ depends on the policy rule parameter F. The correlation of the endogenous variable with the exogenous variable also depends on the policy rule parameter F_z, which may attenuate the sensitivity of the policy target to the exogenous variable z_t. The Lucas critique is a critique of the omission of the reverse causality involved by feedback rule on the estimation of the policy transmission mechanism estimated using the reduced form of "closed-loop" dynamics, and believing that $A+BF$ does not depend on F.

The Lucas critique (1976) does not prove nor state that micro-foundations as a solution to the Lucas critique. Micro-foundations acts only as a reminder that there is an (optimal) negative-feedback rule. But the difficult step is to identify separately A, B and F knowing that there is a reverse causality between the policy instrument and the policy target in the transmission mechanism versus in the policy rule.

In particular, the sign of B is crucial. The price puzzle in vector auto-regressive models is exactly this problem: one obtains the funds rate increases when there is a positive shock on inflation. This corresponds to the intuition of the negative-feedback Taylor rule. Conversely, one obtains that inflation increases following a positive shock on the funds rate, which is the price puzzle: one expect an opposite sign in the transmission mechanism. There is an endogeneity problem because of the reverse causality of the feedback rule.

This reverse causality parameter identification problem is similar to the one of the supply and demand price elasticities with respect to quantities. The supply of goods should increase when price increases, whereas the demand of goods should decrease when price increases, but there is only one covariance for prices and quantities with a unique sign. Then, instrumental variables can be looked for the identification of both parameters (Koopmans (1950)).

The Lucas (1976) critique is a parameter identification problem due to the reverse causality of negative-feedback rules between policy instruments and policy targets. Its answer has to be found in seeking instrumental variables for the identification of parameters, in particular B and F (Koopmans (1950)). A micro-foundation does not necessarily provide these instrumental variables. Seeking instrumental variables is not the objective of theoretical micro-foundations.

2.3 The indeterminacy of a proportional feedback rule

Let's consider that the exogenous forcing variable is set to zero $z_t = 0$. A single policy instrument (Central bank funds rate i_t) reacts to the deviation of a single policy target (inflation π_t) from it set point at date $t = 0$ according to a proportional feedback rule with a given real number for the policy rule parameter F:

$$i_0 = F\pi_0 \text{ with } F \neq 0 \text{ given, } F \in \mathbb{R}^* \tag{5}$$

Both variables are written in deviation of their long run equilibrium. In this static model, a predetermined variable is defined such that its value is a given real number at date 0. A non-predetermined or forward-looking or jump variable is defined such that its value is not given at date 0. A researcher decides arbitrarily if a variable is forward-looking or backward looking.

Proposition 1 *A unique solution for a single policy instrument responding to a single policy target with a proportional feedback rule $i_0 = F\pi_0$ and a given policy rule parameter $F \neq 0$ is obtained when exactly one of the two variables (policy instrument or policy target) is predetermined.*

Proof. *Let us assume a proportional feedback rule $i_0 = F\pi_0$ with $F \neq 0$ given. If the policy target π_0 is given (predetermined) and if the policy instrument i_0 is not predetermined, then $i_0 = F\pi_0$ is the unique solution for the policy instrument. If the policy target π_0 is not predetermined and if the policy instrument i_0 is predetermined, then $\pi_0 = F^{-1}i_0$ is the unique solution for the policy target, because $F \neq 0$. If the policy target π_0 and the policy instrument i_0 are predetermined, there is no solution except if $F = i_0/\pi_0$. This line $i_0 = F\pi_0$ has a zero probability to occur in the continuous plane of the policy instrument and the policy target that can be chosen: $(\pi_0, i_0) \in \mathbb{R}^2$. If the policy target π_0 and the* *policy instrument i_0 are not predetermined, there is an infinity of solution according to the line defined by the proportional policy rule $i_0 = F\pi_0$, which is a subspace of dimension 1 in the plane of the policy instrument and the policy target: $(\pi_0, i_0) \in \mathbb{R}^2$. Table 1 summarizes these results.* ■

Table 1. Unique equilibrium with proportional feedback rule

Policy instrument:	Predetermined policy target π_0 given	Forward-looking policy target $\pi_0 =?$
Predetermined: i_0 given	No solution except if: $\frac{i_0}{\pi_0} = F$	Determinacy: $\pi_0 = F^{-1} i_0$
Forward-looking: $i_0 =?$	Determinacy: $i_0 = F\pi_0$	Infinity of solutions such that $i_0 = F\pi_0$

The arbitrary hypothesis that both the policy targets and the policy instruments are simultaneously jump variables (instead of having one jump variable and one predetermined variable for the policy instrument) is at the origin of the divergence of mainstream macroeconomics from the field of control in the 1990s.

2.4 Negative-feedback rule parameters solution

Either inflation either the funds rate are predetermined and the other one is forward-looking. Negative feedback rule parameters are such that the auto-correlation of the policy target is lower that when there is no feedback $F = 0$ and stable and such that the cross correlation with the exogenous variable is lower than in the case where $F_Z = 0$:

$$D_{nf} = \{F \ / \ 0 \leq \lambda = A + BF < \min(1, A)\}$$
$$0 \leq \mu = C + BF_z \leq C$$
$$\pi_t = \lambda^t \pi_0 + \left(\sum_{k=0}^{k=t-1} \lambda^{t-1-k} \rho^k \right) \mu z_0$$
$$i_0 = F\pi_0 + F_z z_0 \Rightarrow \pi_0 = \frac{1}{F} i_0 - \frac{F_z}{F} z_0 \text{ with } F \neq 0$$

Two roots are strictly below one ($\lambda = A + BF$ and ρ). Transitory dynamics still exist when either z_0 or its auto-correlation ρ are equal to zero: $\pi_t = \lambda^t \pi_0$. Persistence of inflation is endogenous: it is an affine function of the policy rule parameter F measuring the magnitude of the response of the funds rate to inflation. The range of values allowed for F are given by the inequality $0 \leq \lambda = A + BF < \min(1, A)$. Inflation is turned to be stationary after policy intervention if ever it is non-stationary without policy intervention $(A > 1)$.

In case π_0 is forward-looking, it is anchored on the funds rate and the initial shock at the initial date: there is no indeterminacy on the initial condition on inflation. Conversely, if π_0 is predetermined, the initial funds rate is anchored on initial inflation and on the initial value of the shock: there is no indeterminacy.

2.5 Positive-feedback rule parameters solution

Both the funds rate (policy instrument) and inflation (policy target) are jump (forward-looking) variables.

Determinacy implies that they are both collinear with the only predetermined variable z_t. Only one eigenvalue should be stable: as there is already the given stable exogenous persistence of the auto-correlated forcing variables which is stable ($0 \leq \rho < 1$), it should be the case of the endogenous persistence of inflation $\lambda = A + BF$, which restricts values of the policy rule parameter F. In addition, because the policy instrument is restricted to depend only on one variable, this implies an identification restriction on its two policy rule parameters, for example $F_z = 0$.

$$D_{pf} = \{F \ / \ 1 < \lambda = A + BF \text{ or } A + BF < -1\} \tag{6}$$

$$F_z = 0 \tag{7}$$

$$\pi_t = \frac{C}{\rho - (A + BF)} \rho^t z_0 \tag{8}$$

$$i_0 = F\pi_0 = \frac{FC}{\rho - (A + BF)} \rho^t z_0 \tag{9}$$

Linear algebra seeks the slope of the eigenvector related to the stable eigenvalue $\frac{C}{\rho-(A+BF)}$ so that the policy target is a linear function of the forcing variable. Then, the endogenous persistence of the first solution $(A + BF)$ is replaced by an exogenous persistence ρ which does not depend on the policy rule (which would seek inflation to be non-stationary). If the auto-correlation ρ of the shock or if its initial condition is zero, there is no longer transitory dynamics of inflation. The policy rule parameter may tend to infinity, but nonetheless, taking into account the slope of the eigenvector $\frac{C}{\rho-(A+BF)}$, it turns to have a finite value $\frac{-C}{B}\rho^t z_0$.

The exogenous variable is assumed to be not observable in solution 2. If it is observable, for example, oil price, its auto-correlation may not be identical to the auto-correlation of inflation. In this case, it will contradict the prediction of solution 2. For this reason, proponents of solution 2 assume a *non-observable latent* forcing variable z_t. This variable provides an exogenous persistence for inflation which is not controlled by the policy maker's policy rule parameter F. With this assumption solution 2 cannot be falsified contrasting the auto-correlation of the exogenous shock with the auto-correlation of inflation and of the funds rate. It cannot be proven false.

This positive-feedback rule parameters solution is based on the following assumption: when policy targets are forward-looking variables, the policy instrument is arbitrarily assumed to be forward-looking. The negative-feedback set of rule parameters has no intersection with the positive feedback set of rule parameters:

$$D_{nf} = \{F \ / \ 0 \leq \lambda = A + BF < \min(1, A)\} \cap D_{pf} = \{F \ / \ 1 < \lambda = A + BF \text{ or } A + BF < -1\} = \varnothing \tag{10}$$

Proponents of the positive-feedback rule parameters solution often describe it as a negative-feedback stabilizing mechanism. This is a mistake. This linear algebra solution is not compatible with the mechanism of negative-feedback rule parameter. The usual economic explanation of reducing inflation persistence by the policy rule parameter does not hold. The policy rule parameter restrictions destabilizes the transmission mechanism of dimension two, leaving only one stable dimension for the dynamics. However, the policy rule parameter restrictions stabilizing property are not robust to the case where the parameters of the model are not perfectly known, which is the key concern of the field of control.

3 Negative-Feedback Mechanism and Stabilization Policy

3.1 Negative-Feedback Self-Stabilizing Markets

Mayr (1973) explains how Smith (1776) uses the mechanism of negative-feedback in particular in order to describe the local stability of supply and demand equilibrium. In Smith (1776), the negative-feedback comes from the private sector supply.

Smith (1776) anticipates Lyapunov (1898) definition of local stability. However, because of the lack for mathematical formulation which was to come, he provides in words a mechanism close to the pork cycle ((1920)). But not all feedback mechanism are stabilizing: they may maintain the dynamic system to be unstable.

Mayr (1973) also concludes that although Smith knew Watt and engineers technique with negative-feedback, they were not an analogy he used when he explained negative-feedback mechanisms in the Wealth of nations. Mayr (1973) had in mind the close connection which occurred between the trans-disciplinary field of control which emerged in the 1950s to the 1970s. During this period, there is factual evidence that any new technique in the field of control and/or dynamic games was quickly transplanted into economics in say less than three years. Smith (1776) self-stabilizing market with negative feedback on partial equilibrium supply and demand of a given market have been extended to macroeconomic level negative feedback saving or consumption behavior.

Ramsey (1928), Koopmans (1965) and Cass (1965) proposes a negative-feedback mechanism for optimal savings. If the stock of wealth or the stock of capital is below its long run optimal target, consumption decreases proportionally according to a negative feedback proportional rule: $c_t = Fk_t$, where both the consumption flow and the capital stock are written in deviation of their long run target. To become rich, the poor needs to save more. To become poorer, the super-rich needs to save less and consume more that its long run set point of consumption, if the stock of capital is higher than its long run target. The same mechanism applied to a saving account with an average target of the stock of wealth. Kydland and Prescott (1982) stick to this idea, with random productivity shocks deviation below and over target of the stock of capital. Private sector negative feedback determines self stabilizing savings behavior.

This approach makes two assumptions.

Firstly, it is assumed that negative feedback generated *within the private sector* always eliminates non-stationary or diverging and unstable dynamics in the medium run, although they may be short run deviations.

Secondly, even it this is true, it is not considered that there may still be a welfare gain of using an *additional* macroeconomic policy instrument available to macroeconomic policy makers which can control the policy target (for example such as $B \neq 0$ in the scalar order one model). Using a negative-feedback mechanism, a policy maker can then increase the speed of convergence towards long run equilibrium or equivalently decrease the persistence $A + BF$ of the policy target before reaching its long run equilibrium values.

3.2 Classic Control Simple Rules for Stabilization Policy

In classic control, the policy maker decides upon a "desired" speed of convergence of the policy target. For example, he sets a "desired" value $0 \leq \lambda^* = A+BF = 0.6 < 1$. This decision is called in classic control "pole placement", because λ is a pole of the Laplace transform fraction of polynomials of the "closed-loop" system. Then, accordingly, the policy maker decides on a policy rule parameter $F = \frac{\lambda^* - A}{B} = \frac{0.6 - A}{B}$. This policy rule parameter belongs to the negative feedback stability set: $D_{nf} = \{F \text{ such that } 0 < A + BF < \min(1, A)\}$.

Following Lerner (1944) mechanical Keynesian stabilization rules, an electrical engineer, Tustin (1953) devoted several years to apply classic control methods used in electrical systems on Keynesian macro-models. Another former electrical engineer, Phillips (1954, 1957) and in his Ph.D. used proportional, integrated and differentiate (P.I.D.) rules of classic control to stabilize an economic model using negative-feedback mechanism, after having built the water computer Moniac.

Taylor (1968) master thesis merged Phillips (1961) model of cyclical growth with Phillips (1954) P.I.D. rules. During all his life, Taylor remained faithful to the research program of his master thesis: stabilization policy with negative-feedback rules. The Taylor (1993) rule is a proportional feedback rule. The Taylor principle stating that the funds rate should respond by more than one to deviation of inflation from its long run target ($F > 1$) corresponds to the classic control condition for negative-feedback rule parameters such that $0 < A + BF < \min(1, A)$, for models such that $B < 0$ and $A = 1 - B > 1$ (Taylor (1999)):

$$A + BF = 1 - B + BF < 1 \text{ and } B < 0 \Rightarrow F > 1 \tag{11}$$

There is also another condition with an upper bound on F such that $0 \leq A + BF$, where the persistence is equal to zero. The policy maker feedback achieves a complete stabilization of the policy target next period after any shock.

3.3 Kalman, Optimal Control Rules and the Accuracy of Measurement

The emergence of the quadratic loss function is described in Duarte (2009). The quadratic loss certainty equivalence property was found by Simon (1956), Theil (1959) and Malinvaud (1969).

Kalman (1960), an electrical engineer, wrote the key paper for solving linear quadratic optimal control (linear quadratic regulator. Kalman wrote rarely on economics and was relatively skeptical about the contributions of economists. He was invited in the world econometric congress in 1980 in Aix en Provence, but his paper was not published in Econometrica. Kalman received the highest honour of USA science, the US medal of science in 2009.

(1) Kalman (1960) defined controllability which extends the static result by Tinbergen (1954) principle: there should be as many policy instrument than policy targets. Kalman's (1960) controllability definition is such that a single instrument can control for example three policy targets but in three periods. This property is hardly known in macroeconomic dynamics.

(2) Kalman's (1960) linear quadratic regulator sets the solution of stabilization policy facing a quadratic loss function. The crucial role of the quadratic (strict convexity) adjustment cost of the policy instruments in the loss function (for monetary policy, changing

the cost of the policy rate). The larger the cost function, the lower the policy rule parameters without an auto-regressive component in the rule. DARE with no closed form solution

(3) Kalman's (1963) filter estimates recursively the model taking into account at each period new data. The Kalman filter is the dual optimization problem of the linear quadratic regulator. It estimate expectations conditional to new information of the current period. It is currently used for global positioning systems dynamic estimations of locations. For example, a first paper by Muth (1960) updates conditional expectations based on new information: Hansen and Sargent (2007) highlight that it is a particular case of Kalman filter estimation (1963). which is used for example for global positioning systems. The methods of macroeconomics « learning » model consists of variations of Kalman's filter idea (Evans Honkapohja or Kendrick's learning).

(4) Kalman's merged both approached: the measurement of the transmission mechanism with the Kalman filter and the optimal policy based on the LQR using these estimated parameters in the linear quadratic gaussian (LQG) system, which has been unfortunately rarely used by macroeconomists.

(5) Kalman emphasized that the most important issue for stabilization policy is the *accuracy of the measurement* and estimation of "reduced form" parameters of the system of policy transmission mechanism (A and most importantly B and its sign) instead of a quest for an exact theoretical knowledge of the inner working of the transmission mechanism. This quest corresponds to the micro-foundations in macroeconomics.

This history of macroeconomics of the last fifty years reveals that it is much easier to develop a large variety of theoretical micro-founded models conflicting among themselves than to provide accurate measurements of A, B and F. For example, the three editions of Walsh (2012) textbook limit to half a page the description of the price puzzle econometric issue. The price puzzle is such that $B > 0$ and $F > 0$, so that there seems to be positive feedback mechanism on inflation persistence of policy $A + BF > A$. Another half a page of the textbook shows impulse response functions where inflation increases following a rise of the funds rate. The remaining content of the textbook is consists of two to three hundred pages of an accumulation of micro-founded theoretical macroeconomic models which contradict one with the other.

Our knowledge of stabilization policy would be very much different if all those efforts would have been targeted to the accuracy of the measurement of the sign of the cross variables partial correlations (in matrix $\mathbf{B}$) of the various transmission mechanisms of stabilization policy instead of building more and more micro-founded theoretical models. But these parameters are much more difficult to identify than writing micro-founded macro-models which can be observationally equivalent, with under-identified or weakly identified parameters, which can hardly be proven false, and allows controversies, citations and fame within the field of macroeconomics.

3.4 From simple rules to optimal policy rules

In classic control ("simple rules" in macroeconomics), the policy maker targets a value of persistence of the policy targets ($0 < \lambda^* = A + BF^* < 1$). Kalman's optimal control grounds this choice based on a quadratic loss function possibly discounted by a factor β, with a non-zero relative quadratic cost of changing the policy instrument $\alpha > 0$:

$$\sum_{t=0}^{+\infty} \beta^t \left(\pi_t^2 + \alpha i_t^2\right) \text{ with } \quad > 0 \text{ and } 0 < \beta \leq 1 \tag{12}$$

subject to the same transmission mechanism than the one of classic control. For a given transmission parameters A and B, and for a given discount factor β, there is a *monotone* relation between the cost of changing the policy instrument α (the policy maker's preference) and the optimal policy rule $F^* (\alpha)$ and the optimal persistence $\lambda^* (\alpha) = A + BF^* (\alpha)$. If one can identify and estimate A, B and $F (\alpha)$, if one sets an identification restriction on β, one can then identify and estimate the policy makers preference α. Therefore a reduced form of a simple rule parameter $F (\alpha)$ corresponds to an optimal rule with these preferences α. There is a policy maker preferences which "rationalize" an estimated value of a simple rule parameter F to be a reduced form of an optimal rule parameter $F (\alpha)$.

Hence, there is no *contradiction* between classic control and optimal control. There is no reason to claim that "simple rule" should be preferred to "optimal rules", because both are observationally equivalent, with the same predictions and policy maker behavior. Optimal control is filling a gap in classic control "pole placement" method which is: how do you decide upon the persistence λ^* of the policy target.

3.5 Stackelberg dynamic games and stochastic replanning

Prescott (1977) propagated a false commonplace: macroeconomists are unique because stabilization policy is not the usual game against nature done by engineers. This is the reason why engineer' control should not be used by policy makers.

This is false. The field of optimal control was subsidized by the army in the 1940s and the cold war. War is anything but a strategic game, so the concern was not limited to games against nature. Dynamic Nash games emerged in applied mathematics in the field of control, applied mathematics and engineering departments. In particular, Isaacs (1965) book, still in press, summarized the literature on Nash dynamic games, which built on Von Neumann and Morgenstern (1944). Economist were latecomers in the field of dynamic games which evolved in the field of control. As usual, until the 1980s, transplants into economics of news tools of control were achieved in less that three years.

In other engineering departments Simaan and Cruz (1973a,b) were the first to investigate Stackelberg dynamic games and to demonstrate that their time-inconsistency result. Once known, this result was included in Kydland PhD. Kydland (1975) first publication is a direct economic application of Simaan and Cruz (1973b) result. Kydland and Prescott (1980) transplanted again Simaan and Cruz (1973a) time-inconsistency into economics.

For economists, this amounts to solve a dynamic model of dynamic Ramsey (1927) static optimal (tax) policy in a dynamic context were the follower does intertemporal optimal control optimization, such as Ramsey (1928) optimal savings. Hence, the label of "*Ramsey optimal policy (under commitment)*" has been assigned for Stackelberg dynamic games in macroeconomics, although the formal credit is for Simaan and Cruz.

Stackelberg dynamic games are used with quadratic loss function for the leader and the follower. They are extensions of Kalman's linear quadratic regulator solution. The leader takes into account the follower marginal conditions. However, like the tower of control interacting with the pilot of an airplane, they can alter the behavior of the decision variables of the pilot or follower. These decision variables are jump variables. Hence, the policy maker can anchor the initial value of these jump variables optimally, minimizing the loss function with respect to the follower's decision variable at the initial date.

In our example, let us assume that prices and inflation are private sector's "jump" decision variable, with unknown initial condition, instead of assuming a given initial value π_0. This initial condition is optimally decided by the policy maker, where the anchor parameter $P^*(\alpha)$ of the jump variable on predetermined variable is derived from optimization. It depends on the policy maker's preferences:

$$\left(\frac{\partial L}{\partial \pi}\right)_{t=0} = 0 \Rightarrow \pi_0^* = P^*(\alpha) z_0 \tag{13}$$

Simaan and Cruz (1973b) result is that if the policy maker re-optimises next period using the same formula, it contradicts the optimal plan for inflation, which would for example depends on past values of the jump variable (with parameters G and H): $\pi_1^* = G(\alpha)\pi_0^* + H(\alpha) z_1 \neq P^*(\alpha) z_1$.

However, the gap between the previous period plan and the new optimal plan is negligible for very low or very large cost of changing the policy instrument α. In these cases, there is a negligible cost of time-inconsistency, although it is still logically feasible.

This time-inconsistency results holds without any change of the transmission mechanism nor any change of policy makers and private sector preferences. This result is distinct from the commonly held view of time-inconsistency which assumes that the transmission mechanism and/or the preferences of the agents changed over time.

Prescott (1977) put forward Simaan and Cruz (1973b) as the proof of the logical inconsistency of any stabilization policy. Hence, one should only rely on self-stabilizing negative feedback forces operating in the private sector. He added this new argument to Smith (1776) statement.

Roberds (1987) provided an answer to Prescott (1977) radical views. He added the assumption agents have a non-zero probability to renege commitment, which measured a partial commitment leading to a partial credibility. In addition, a finite horizon for the commitment, which is part of several democratic institutions with fixed term mandate, is a mean to secure the gains of commitment, which allowing predictable shifts of preferences with a known term.

Prescott's extreme description of policy maker's discretion turned to be a zero-probability case. The policy maker's should continuously erode his own commitment at each instant, with perpetual and instantaneous continuous re-optimization. It assumes an incredible amount of effort by members of an institution in order to kill itself. The logical inconsistency is then in Prescott's (1977) statement.

The literature shifted to model of central bank independence in order to increase the credibility of their commitment. This makes sense if one believes that central banks are useful for stabilization policy and one should not close them.

3.6 Robust Control Rules

The third step after classic control and modern optimal control has been robust control (Doyle et al. (1996), Hansen and Sargent (2007)) which emerged around 1978. Doyle (1978) noticed that Kalman linear quadratic Gaussian model has not enough margins with respect to instability ($A + BF > 1$) when there was uncertainty on the transmission parameters.

Robust control assumes that the knowledge of the transmission parameters is quite uncertain but on a known *finite interval* and not up to the point that we do not know the signs of these parameters. In our example, it may correspond to $B_{\min} < B < B_{\max} < 0$. Then, a dynamic game is assumed between the policy maker and an evil agents who tries to fool as much as possible the policy maker. The policy maker is minimizing the maximum of the losses in the range of uncertainty on the parameters. Again, Kalman's solutions of the linear quadratic regulator turned to be instrumental in the solutions of the tools of control robust to misspecification.

In economics, the uncertainty on parameters was put forward by Brainard (1967). Very soon after robust control tools emerged, Van der Muehlen (1982) wrote the first working paper in economics using them. Since the 1990s, Hansen and Sargent promote the transplant of robust control tools into macroeconomics, based on Doyle et al. (1995) and Whittle (1995).

4 Stabilization Policy Ineffectiveness

Monetary policy and budgetary policy ineffectiveness was a hot topic in the 1970s (Prescott (1977), Currie (1985) and Turnovsky (2011)). To give some credit to the tenant of this view (Lucas, Prescott), they did the Ph.D. and their first research in the 1960s, with full employment, more than 4% growth rate of GDP per year, international control of capital flows and no banking crisis. It may well be the case that stabilization monetary and fiscal policy was a second order effect at this time. Unfortunately for the tenant of the policy ineffectiveness debate, the fact that inflation fell like a stone following Volcker and Thatcher monetary policy in the early eighties was an obvious contradiction of policy ineffectiveness.

We order the arguments against policy effectiveness firstly with respect to policy makers preferences parameters and secondly with respect to the parameters of the policy transmission mechanism.

Argument 1: Government loss function includes a large relative weight for rent seeking with a negligible relative weight for the benevolent Ramsey planner loss function of a benevolent Ramsey planner. The feedback policy rule of control may work, but it is not used properly because of rent-seeking, so that the public choice cost-benefits optimum is to close central banks.

Argument 2: The relative cost of changing policy instruments is very high because of several constraints: zero lower bounds of Central Bank policy rate, public debt sustainability, threat of speculation crisis against the sustainability of an exchange rate peg, lack of coordination of international policy with beggar-thy-neighbour policy, international competition on the goods, labour and financial markets.

Argument 3: The relative cost of economic fluctuations is negligible, because of the low volatility of policy targets (unemployment, output gap, inflation, credit, asset prices) in the case without policy intervention.

Argument 4: The policy transmission mechanism is such that the private sector economic system is not Kalman controllable. Fiscal and monetary policy have negligible direct and indirect effects on output, inflation, asset prices, money and credit. For example, the monetary policy short run interest rate has no effect on long run interest rate, because of preferred habitat hypothesis. Although long term rates decrease asset prices, short term rate have then no effect on asset prices. Ricardian equivalence is another example, with a zero sensitivity of consumption to changes of lump-sum taxes.

Argument 5: The private sector economic system may be Kalman controllable but there is uncertainty on the signs of the cross-correlations of the policy transmission. The distribution of the multiplicative parameter of the policy transmission is centered in zero, symmetric, with a large standard error. If the distribution of the signs of the effects is symmetric.

Argument 6: The specific time-inconsistency issue of Stackelberg dynamic games cannot be fixed, because governments are never able to commit (Prescott (1977)). But Thatcher 1979-1980 and Volcker's disinflationary monetary policy were evidence that policy makers can successfully commit.

5 Positive Feedback Rule Parameters

5.1 From Vaughan (1970) to Blanchard and Kahn (1980)

Vaughan (1970) proposed a solution of Kalman's linear quadratic regulator using Jordan decomposition of the Hamiltonian matrix. The aim is to control n state variables with a known policy transmission mechanism including n equations. This solution uses n jump co-state variables which are Lagrange multipliers related to n first order Euler equations. The combination of the n first order Euler conditions and of the n equations of the transmission mechanism is an Hamiltonian linear system with a square matrix $\mathbf{H}_{2n}$. This matrix is symplectic: its transpose is equal to its inverse. Hence, it includes n stable roots and n unstable roots of its characteristic polynomial. The co-state variables are linear functions of the decision jump variables of the optimizing agent.

Kalman unique optimal policy rule $i_t = F^* (\alpha) \pi_t$ is then found as a constraint so that dynamics of the Hamiltonian system remains in its unique stable subspace of dimension n. This is equivalent to Kalman (1960) unique solution of a matrix Riccati equation for solving the linear quadratic regulator. This rule stabilizes the dynamics of the n state variables. The Hamiltonian system includes n fictitious additional dimensions related to co-state variables, but these n additional dimensions are only a computational artefact of Lagrange solution. The fact that an Hamiltonian system includes n unstable eigenvalues in addition to n stable eigenvalue does not imply the local instability of the dynamics of the n state variables.

Blanchard and Kahn (1980) were seeking the solutions of linear dynamic models with rational expectations which are not based on optimal intertemporal behavior. Hence, their dynamics are arbitrarily chosen by the researcher as a real square matrix $\mathbf{A}_{\overline{n}+\overline{m}}$ which is not necessarily symplectic like the Hamiltonian matrix $\mathbf{H}_{2n}$. The matrix $\mathbf{A}_{\overline{n}+\overline{m}}$ has an arbitrary number $\overline{n}$ of stable roots of its characteristic polynomial and an arbitrary number of $\overline{m}$ unstable root. By contrast to the Hamiltonian, one does not have necessarily $\overline{n} = \overline{m}$.

A jump variable is defined by a mathematical criterion in intertemporal optimization solution using Lagrange multipliers. By contrast, in Blanchard and Kahn (1980), jump variables are renamed "forward-looking variables" and they are chosen *without a mathematical criterion* (arbitrarily) by the researcher. Hence, their number is decided to be equal to m, which is not necessarily equal to the number of unstable roots $\overline{m}$. The remaining $n = \overline{n} + \overline{m} - m$ variables are chosen to be "predetermined" without a mathematical criterion by the researcher.

The uniqueness of the solution is obtained by the determinacy condition such that $\overline{m} = m$ with implies that $\overline{n} = n$. Hence, one can apply Vaughan solution such that forward-looking variables are a unique linear combination of predetermined variables, like Vaughan's unique optimal policy rule solution. Blanchard and Kahn (1980) solution is the key characteristic of rational expectations models.

Blanchard and Kahn (1980) acknowledged that their determinacy condition is always satisfied when there is intertemporal optimisation, citing Vaughan (1970). In the 1970s, most of rational expectations models were not based on optimal intertemporal behavior. In the 1990s, mainstream macroeconomic models shifted to optimal intertemporal behavior, so that Blanchard and Kahn (1980) is useless, and Kalman (1960) and Vaughan (1970) unique solution applies for linear quadratic models.

Table 2

Authors	Vaughan (1970)	Blanchard Kahn (1980)
Model	Linear quadratic regulator	No optimization
Variables	n state variables CHOSEN	n predetermined CHOSEN
Variables	n jump costates (instruments) Euler equations DEDUCTED	m forward-looking CHOSEN
Matrix	**H:** symplectic with Euler equations	**A**: any ad hoc values
Stable roots: $\lvert\rho_i\rvert < 1$	n	$\overline{n}$
Unstable roots: $\lvert\rho_i\rvert > 1$	n	$\overline{m}$
Determinacy	Always	$n = \overline{n}$, $m = \overline{m}$

5.2 Assuming policy instruments and targets are simultaneously jump variables

Currie and Levine (1985) included policy maker's proportional feedback rule $i_t = F\pi_t$ in linear rational expectations models without intertemporal optimization. They assumed implicitly that policy instruments and policy targets are simultaneously jump variables. They applied mechanically Blanchard and Kahn (1980) determinacy condition. This led them to use the solution of positive-feedback rule parameter presented in a previous section.

They also defined "optimal simple rule" where a loss function is minimized subject to dynamics constrained by positive-feedback rule parameters. This definition was later applied to DSGE models using simulation grids for policy rule parameters by Schmitt-Grohé and Uribe (2007).

These papers were applied to models not derived from inter-temporal optimization of the coordination of optimal policy between two open economy countries. They included exchange rate and the price of consumer good as jump forward-looking variables. Later, Currie and Levine (1993) published a book including several of their papers. Up to 1993, they were not cited except by themselves.

5.3 Multiple equilibria with positive feedback "active policy"

Leeper (1991) used a simplified Ramsey (1928) intertemporal optimal saving model and two proportional feedback rules: an interest rate rule and a tax rule responding public debt deviation.

$$i_t = F_\pi \pi_t \text{ and } \tau_t = G_b b_t \tag{14}$$

The first equation is a Fisher real interest rate equation with a constant "real" discount rate $\rho > 0$:

$$E_t \pi_{t+1} = \beta i_t = \beta F_\pi \pi_t \tag{15}$$

The second equation is public debt accumulation:

$$b_{t+1} = \left(\frac{1}{\beta} - G_b\right) b_t + aE_t \pi_{t+1} + b\pi_t \tag{16}$$

Inflation, interest rate and taxes are assumed to be jump (forward-looking) variables. Public debt is the only predetermined variables so that there should be one stable root for determinacy. But the dynamic system includes two roots to stabilize, so that there is a choice either to stabilize the "inflation root" and destabilize "public debt root" (fiscal theory of the price level equilibrium) or to destabilize the "inflation root" and stabilize "public debt root".

Two equilibria exist because at least one jump variable and at least one predetermined variable are both controllable by policy maker's instrument. In the transmission mechanism, there is a non-zero correlation between each of these policy targets and at least one policy instrument. Hence, there are two eigenvalues which are under control by the policy-maker.

Leeper (1991) defines an "active" monetary policy such that the eigenvalue $|F_\pi \beta| = |\lambda_\pi| > 1$ (or $|F_\pi| > 1$ for the linearized version), whereas in optimal control, an active policy is such that the root $\lambda = A + BF$ are stable $|\lambda| < 1$.

The fiscal theory of the price level faced its first ten years 1991-2001 under debate. Until 2001, around ten economists cited Leeper, including Woodford (1993), Sims (1998), Cochrane (1998). Several other macroeconomists, e.g. Buiter (2001), criticized the predictions of the fiscal theory of the price level, in particular public debt was an anchor for inflation.

5.4 The new-Keynesian model

During the period 1995-1999 emerged the new-Keynesian model which added Calvo price rigidity (the new-Keynesian Phillips curve) to Leeper (1991) model with an Euler consumption equation instead of a Fisher equation (King and Kerr (1995), Clarida, Gali and Gertler (1999), Woodford (1999)). Derived from intertemporal optimization, private sector variables inflation, output gap are jump decision variables. The new Keynesian model included a feedback proportional interest rule now labeled Taylor (1993) rule. As in Leeper, it assumed that the funds is a jump variable as well as the policy targets.

The way out of the fiscal theory of the price level controversy was to assume that there is zero net supply of public debt (Gali (2015), footnote 3). Then, public appears temporarily at the beginning of the intertemporal optimization of the representative consumer. But this consumer is myopic: he does not notice that there is no store of value for his wealth as public debt. Then, an equilibrium occurs with zero net supply of public debt. Public debt is therefore ruled out of the model. This eliminates the fiscal theory of the price level second equilibrium and the related controversy.

However, the model would then be a degenerate rational expectations model with no predetermined variables, so that determinacy is achieved by instantaneous jumps to the long run equilibrium of funds rate, inflation and output gap for any transitory shocks. There will be never transitory dynamics. In order to circumvent this outcome, public debt predetermined variable is replaced by exogenous non-controllable auto-regressive forcing variables (cost-push shock and consumption shock). Their root is the auto-correlation parameters which are stable and cannot be modified by policy rule parameters. This assumption allows transitory dynamics driven by these exogenous auto-correlation of shocks.

The consumption and output gap Euler equation implies a positive correlation between future output gap and the current value of the funds rate. This is the opposite sign of the transmission mechanism of Keynesian models. The new-Keynesian Phillips curve implies a negative correlation between future inflation and the current value of the output gap. This is the opposite sign of the transmission mechanism of the accelerationist Phillips curve. Negative-feedback rule parameter is such that if current output gap is negative, funds rate should increase ($F_x < 0$), because of the positive correlation with funds rate and future consumption in the transmission mechanism (Euler consumption equation with intertemporal substitution effect):

$$i_t = F_\pi \pi_t + F_x x_t \tag{17}$$

$$\underbrace{x_t \downarrow \Leftrightarrow i_t \uparrow}_{\text{Feedback rule}} \underbrace{\Leftrightarrow x_{t+1} \uparrow \Leftrightarrow \pi_{t+2} \downarrow}_{\text{Transmission mechanism}} \tag{18}$$

By contrast, a Keynesian cost of capital transmission mechanism has the opposite sign of correlations than an intertemporal substitution effect. A rise of current funds rate leads to a decrease of future output which then depresses inflation. Accordingly, the negative-feedback rule parameter F_x should be positive:

$$\underbrace{x_t \uparrow \Leftrightarrow i_t \uparrow}_{\text{Feedback rule}} \underbrace{\Leftrightarrow x_{t+1} \downarrow \Leftrightarrow \pi_{t+2} \downarrow}_{\text{Transmission mechanism}} \tag{19}$$

The condition for $F_\pi > 1$ is required for negative-feedback rule parameters, which is called the new-Keynesian Taylor principle.

However, if one assumes that funds rate is a jump variable as well as output gap and inflation, the positive-feedback rule parameter solution allows $F_x > 0$ and $F_\pi > 1$. Hence, one finds the same sign of rule parameter $F_x > 0$ with the intertemporal substitution transmission correlation and positive-feedback rule parameters than in the case with the cost of capital effect transmission correlation and negative-feedback rule parameters.

Some macroeconomists state that the sign of correlation between future values and current values is irrelevant because the causality goes from expectations to current values. But what matters in the transmission mechanism is the correlation, not the causality from current values to expectations.

Conversely, there is no mathematical demonstration that the policy instrument is a jump variable when policy targets are jump variables.

5.5 Doublethink with Ramsey optimal policy under commitment

Ramsey optimal policy under commitment or quasi-commitment use a mathematical optimal condition to find an optimal anchor of forward-looking policy target on predetermined policy instruments. As it is a benchmark in terms of welfare and optimal behavior of policy makers, new-Keynesian authors bargained with mathematics in order to state that Ramsey optimal policy with its negative-feedback rule parameter was "nearly" the same than positive-feedback rule parameters solution (Woodford (1999)). Impulse responses can be close, although surrounded by unstable paths in the positive-feedback rule parameter case versus surrounded by stable paths in the negative-feedback case. For some parameter values, it may be possible to approximate Ramsey optimal policy impulse response by optimal simple rule.

Hence, the idea has been to focus on impulses responses and to neglect the roots of the characteristic polynomial and the related policy rule parameters. For example, instead of writing Ramsey optimal policy rule responding to policy targets deviations, it has been written as a function of Lagrange multipliers.

By the truncation and the selection bias of the mathematical results, this approach suggests that there is seemingly no inconsistency between positive-feedback rule parameter solution and negative-feedback rule parameter solution. Having policy instrument predetermined or jump variable leads is equivalent. *"Doublethink means the power of holding two contradictory beliefs in one's mind simultaneously, and accepting both of them... In philosophy, or religion, or ethics, or politics, two and two might make five, but when one was designing a gun or an aeroplane they had to make four."* (Orwell (1949)).

6 Conclusion

When losing control, macroeconomics gained new puzzles, new paradoxes, new problems, new inconsistencies and new controversies:

Lack of robustness to misspecification: Giordani and Soderlind (2006) emphasized that the positive-feedback rule parameter solution is not robust to imperfect knowledge of the parameters of transmission mechanism, when using robust control methods. This is not a surprise, as the aim of the negative-feedback mechanism is to stabilize the n dimensions of the n policy targets dynamics, not strictly less than n as with the positive-feedback rule parameters solution. However, this paper had no impact on the field.

Parameter identification problems. One cannot empirically dismiss easily the positive-feedback rule parameter solution, which is fighting fire with fire. This is not easy because of the parameter identification problem due to the reverse causality of negative-feedback rule parameters. Farmer (2007) highlighted the observational equivalence between indeterminacy and determinacy when one adds a lag (which adds a stable root in the characteristic polynomial) in one model with respect to the other model. Cochrane (2011) highlighted that there was under-identification of policy rule parameters for small size model with the positive-feedback rule parameters solution.

Multiple equilibria. The multiple equilibria solutions are usually circumvented by replacing all controllable and endogenous stock of public and private debt and stock of capital by exogenous auto-regressive shocks.

Lack of microfoundations. The lack of microfoundations is crucial for assuming that the policy instrument is also a jump variable when policy targets are a jump variable.

For opportunistic researchers seeking novelty for publications, losing control has been a gold mine. The negative-feedback mechanism which prevailed before 1990 has been reversed. Hence, all papers on stabilization policy published before 1990 were depreciated, replaced by several thousands of publications with positive-feedback rule parameter stabilizing the economy. For the general public as well as for practitioners of policy making, it may be that macroeconomics has gone backwards when losing control.

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